

WhatsApp X Yth. Bpk/Ibu Abdulmuhih di S X Inbox - abdulmuhih1979@gmail X International Journal of Advanc X Download file | iLovePDF X +

← → ↻ ↗ https://mail.google.com/mail/u/1/#search/senatik/FMfcgwxwKjTVTHzMrkooswPHPDdpWNtMM ... ☆

☰ Gmail senatik ? ⚙️ ⋮ UNJSA

Tulis

Kotak Masuk 2  
Berbintang  
Ditunda  
Terkirim  
Draf  
Selengkapnya


Meet  
Rapat baru  
Rapat saya

Hangout  
Abdul Muhith +

Tidak ada chat terbaru  
Mulai yang baru

1 dari 1 < >

Yth. Bpk/Ibu Abdulmuhih di **SENATIK**, Translate dan Proofreading Gratis untuk Jurnal Scopus Kotak Masuk x

 **linda@goodlingua.info** Rab, 18 Nov 2020 08.49 ☆ ↶ ⋮  
kepada saya ▾

Indonesia > Esti ▾ Terjemahkan pesan Nonaktifkan untuk: Indonesia x




Yth. Bpk/Ibu Abdulmuhih,

Assalamualaikum, selamat pagi.

Bpk/Ibu Abdulmuhih, Perkenalkan nama saya Linda, Admin dari **GoodLingua**. Sebelumnya mohon maaf mengirimkan email ini. Kami mendapatkan kontak bapak dari artikel yang di publish di IOP Conference dalam conference **SENATIK**. GoodLingua perusahaan khusus translate academic article seperti journal dan conference paper. Website kami di [www.goodlingua.com](http://www.goodlingua.com)

di Bulan November ini kami ingin menawarkan translate atau proofreading gratis untuk Abstrak khusus jurnal atau conference untuk Bpk/Ibu Abdulmuhih. Abstrak bisa dikirim melalui WA 0856-9355-4284.

Sebagai informasi, **GoodLingua** sudah banyak membantu penulis untuk menerbitkan karya tulis ilmiahnya ke jurnal-jurnal scopus, dari Q4 sampai Q1. berikut beberapa portfolio kami.

Windows e X N W P Chrome O PDF GEOTHERMICS Journal of Energy Institute 10:12 PM 3/15/2021

SENATIK V Prodi Pendidikan Matematika  
FPMIPATI-UNIVERSITAS PGRI SEMARANG  
Jl. Dr. Cipto –Lontar No 1 Semarang  
HP.0858-6512-7933;02857-5713-3157  
Email: [senatik@upgris.ac.id](mailto:senatik@upgris.ac.id)

## **KWITANSI**

N0.

Telah terima Abdul Muhiht

Uang sejumlah Tiga Juta rupiah

Untuk pembayaran Artikel IOP Kode 962 dengan judul *The effects of drugs in chemotherapy as optimal control of tumor growth dynamical model* pada Senatik V

Rp. 3.000.000,-

Semarang, 11 Agustus 2020  
Yang menerima,



  
Rina Dwi S, M.Pd.



**PANITIA SEMINAR NASIONAL  
MATEMATIKA DAN PENDIDIKAN MATEMATIKA (SENATIK) 2020  
PROGRAM STUDI PENDIDIKAN MATEMATIKA FPMIPATI  
UNIVERSITAS PGRI SEMARANG**

No. : 224/Pan.SENATIK/P.Mat/IX/2020  
Lamp. : -  
Hal : *Letter of Acceptance* (LOA)

Semarang, 11 September 2020

Kepada  
Yth. Dinita Rahmalia  
Pemakalah Seminar Nasional Matematika dan Pendidikan  
Matematika (SENATIK) V Universitas PGRI Semarang  
di Tempat

Dengan hormat,

Berdasarkan hasil review *full paper* yang telah dilakukan oleh tim reviewer dan editor, maka panitia Seminar Nasional Matematika dan Pendidikan Matematika (SENATIK) 2020 memutuskan artikel dengan

Judul : *The effects of drugs in chemotherapy as optimal control of tumor growth dynamical model*

ID : 962

Nama Penulis : A Muhith, D Rahmalia and T Herlambang

Telah diperiksa dan diterima untuk disubmit ke *IOP Conference Series: Journal of Physics* yang terindeks Scopus dan Web of Science. Terkait dengan hal tersebut, penulis diminta untuk mengunduh *JPCS Copyright Form* melalui laman <http://bit.ly/CF-senatik> kemudian mengisinya dengan tulisan tangan, discan dan dikirim kembali ke email [senatik@upgris.ac.id](mailto:senatik@upgris.ac.id) hingga tanggal 15 September 2020.

Demikian surat keterangan ini dibuat untuk dapat dipergunakan sebagaimana mestinya. Kami mengucapkan terimakasih atas perhatian dan partisipasinya dalam Seminar Nasional Matematika dan Pendidikan Matematika (SENATIK) V Tahun 2020.

Mengetahui,  
Ketua Program Studi  
Pendidikan Matematika

  
Dr. Lilik Ariyanto, M.Pd.  
NPP. 088602194

Semarang, 11 September 2020  
Ketua Pelaksana



  
Noviana Dini Rahmawati, S.Pd., M.Pd.  
NPP. 118701355

PAPER • OPEN ACCESS

## The effects of drugs in chemotherapy as optimal control of tumor growth dynamical model

To cite this article: A Muhith *et al* 2020 *J. Phys.: Conf. Ser.* **1663** 012006

View the [article online](#) for updates and enhancements.



**IOP | ebooks™**

Bringing together innovative digital publishing with leading authors from the global scientific community.

Start exploring the collection—download the first chapter of every title for free.

# The effects of drugs in chemotherapy as optimal control of tumor growth dynamical model

A Muhith<sup>1\*</sup>, D Rahmalia<sup>2</sup> and T Herlambang<sup>3</sup>

<sup>1</sup> Department of Nursing, Universitas Nahdlatul Ulama Surabaya, Indonesia

<sup>2</sup> Department of Mathematics, Universitas Islam Darul Ulum Lamongan, Indonesia

<sup>3</sup> Department of Information System, Universitas Nahdlatul Ulama Surabaya, Indonesia

\*Corresponding author's e-mail: [abduhmuhith@unusa.ac.id](mailto:abduhmuhith@unusa.ac.id)

**Abstract.** Cancer is the disease caused by disordered hormone so that it causes the lumps to grow abnormally on body tissue, and it is known as a malignant tumor. Some mortalities in the world are caused by cancer, while in Indonesia, cancer contributes to the third-largest death. This research will explain about stability and optimal control of tumor growth dynamical model by drugs in chemotherapy. In the tumor growth dynamical model, there are normal cells, tumor cells, and immune cells. From the mathematical model of tumor growth, some equilibrium points will be analyzed for their stability using eigenvalue. In this research, from the mathematical model of tumor growth, it will be added control, such as drugs in chemotherapy. The method used for solving optimal control problems and resulting numerical solutions is Forward Backward Sweep Method. Based on simulation results, drugs in chemotherapy give effects in a normal cell, tumor cell, and immune cell.

## 1. Introduction

Cancer is the disease caused by disordered hormone so that it causes the lumps to grow abnormally on body tissue, and it is known as a malignant tumor. About 16.65 percent, the mortality in the world is caused by cancer. In Indonesia, cancer contributes to the third-largest death. Unhealthy diet and life, smoke are dominant factors causing cancer. Cancer occurs when normal cells become cancer cells through the mutation process or abnormal growth. Some treatments for reducing cancer have been applied, such as chemotherapy and traditional drugs.

There are many diseases which have been constructed to mathematical model such as influenza, bird flu, dengue fever [1][2][3][4]. In the mathematical model of disease spread, generally, there are susceptible population, infected population, and recovered population [5][6]. From three populations, they can be determined by reproduction number based on available parameters for determining the stability. Besides that, the predator-prey model has also been developed for stability in natural selection [7].

This research will explain about stability and optimal control of tumor growth dynamical model by drugs in chemotherapy. In the tumor growth dynamical model, there are normal cells, tumor cells, and immune cells [8]. From the mathematical model of tumor growth, there are some equilibrium points that will be analyzed their stability using eigenvalue. In this research, from the mathematical model of tumor growth, it will be added control, such as drugs in chemotherapy for reducing the number of tumor cells.



However, in practice, this process also affects normal cell and immune cells so that normal cell and immune cells are also reduced. The usage of drugs in chemotherapy should be proportional. Fewer drugs cause tumor cells to stay grow, and over drugs cause expensive cost and bad for the body tissue.

The method used for solving optimal control problems and resulting numerical solutions is Forward Backward Sweep Method [9]. This method uses state variables with initial conditions and adjoint variables with the final condition in its computation [10]. Based on simulation results, drugs in chemotherapy give effects in normal cells, tumor cells, and immune cells.

## 2. Methods

In the mathematical model of tumor growth, there are three populations included in the system, such as normal cell, tumor cell, and immune cell [8]. It is assumed that normal cell and tumor cells grow based on logistic function and immune cells grow continuously. This model uses the predator-prey model concept in tumor cells and immune cells.

### 2.1. Mathematical model of tumor growth

The mathematical model of tumor growth is presented in equation (1), (2) and (3) [8] :

$$\dot{N} = r_2 N(1 - b_2 N) - c_4 N \quad (1)$$

$$\dot{T} = r_1 T(1 - b_1 T) - \frac{\rho IT}{\alpha + T_0} - c_2 IT + c_3 N \quad (2)$$

$$\dot{I} = s + d_2 \left( \frac{\rho IT}{\alpha + T_0} \right) - c_1 IT - d_1 I \quad (3)$$

Note that the denominator  $\alpha + T_0$  is assumed constant with  $N(t)$  is the population of a normal cell,  $T(t)$  is the population of tumor cell, and  $I(t)$  is the population of immune cell. The other parameters can be seen in table 1.

**Table 1.** The parameter used in the mathematical model of tumor growth.

Notation	Description
$r_1$	Intrinsic rate (per capita growth) of tumor cell growth
$r_2$	Intrinsic rate (per capita growth) of normal cell growth
$b_1$	Carrying capacity of the tumor cell population
$b_2$	Carrying capacity of the normal cell population
$\rho$	Search rate of tumor cell by the immune cell
$d_1$	The natural death rate of immune cell
$d_2$	Conversion factors
$\alpha$	Immune threshold rate
$s$	The growth rate of immune cell (constant)
$c_1$	Coefficient of an inactive immune cell due to interaction with tumor cell
$c_2$	Coefficient of dead tumor cell due to interaction with immune cell
$c_3$	The rate of increasing tumor cell due to normal cell mutation to tumor cell
$c_4$	The rate of decreasing normal cell due to normal cell mutation to tumor cell

Consider the population of tumor cells and immune cells. From the model in equation (2) and equation (3), without the existence of immune cells, the tumor cells grow based on logistic function, and without

the existence of tumor cells, the immune cells grow constantly. When immune cells eat tumor cells, immune cells duplicate their selves for attacking tumor cells.

## 2.2. Existence of solution

The solutions to this problem can be said to exist if  $N(t) \geq 0, T(t) \geq 0, I(t) \geq 0$ . The equilibrium points also should satisfy these conditions [11].

## 2.3. Equilibrium point

The equilibrium points can be determined by  $\dot{N}=0, \dot{T}=0, \dot{I}=0$  in equation (4), equation (5), and equation (6).

$$r_2 N(1-b_2 N) - c_4 N = 0 \quad (4)$$

$$r_1 T(1-b_1 T) - \frac{\rho IT}{\alpha + T_0} - c_2 IT + c_3 N = 0 \quad (5)$$

$$s + d_2 \left( \frac{\rho IT}{\alpha + T_0} \right) - c_1 IT - d_1 I = 0 \quad (6)$$

From equation (4), equation (5), and equation (6), we obtain three equilibrium points:

1. Equilibrium point 1:  $N_{e1} = 0, T_{e1} = 0, I_{e1} = \frac{s}{d_1}$

2. Equilibrium point 2:  $N_{e2} = 0, T_{e2} = \frac{1}{b_1} - \frac{\rho I_{e2}^+}{(\alpha + T_0)r_1 b_1} - \frac{c_2 I_{e2}^+}{r_1 b_1}, I_{e2} = I_{e2}^+$  with  $I_{e2}^+$  is the positive solution of:

$$s + \left( \frac{1}{b_1} - \frac{\rho I_{e2}^+}{(\alpha + T_0)r_1 b_1} - \frac{c_2 I_{e2}^+}{r_1 b_1} \right) \left( d_2 \left( \frac{\rho I_{e2}^+}{\alpha + T_0} \right) - c_1 I_{e2}^+ \right) - d_1 I_{e2}^+ = 0$$

3. Equilibrium point 3:  $N_{e3} = \frac{1}{b_2} - \frac{c_4}{b_2 r_2}, T_{e3} = T_{e3}^+, I_{e1} = \frac{-r_1 T_{e3}^+(1-b_1 T_{e3}^+) - c_3 \left( \frac{1}{b_2} - \frac{c_4}{r_2 b_2} \right)}{\frac{-\rho T_{e3}^+}{\alpha + T_0} - c_2 T_{e3}^+}$  with

$T_{e3}^+$  is the positive solution of :

$$s + \left( \frac{-r_1 T_{e3}^+(1-b_1 T_{e3}^+) - c_3 \left( \frac{1}{b_2} - \frac{c_4}{r_2 b_2} \right)}{\frac{-\rho T_{e3}^+}{\alpha + T_0} - c_2 T_{e3}^+} \right) \left( d_2 \left( \frac{\rho T_{e3}^+}{\alpha + T_0} \right) - c_1 T_{e3}^+ - d_1 \right) = 0$$

Because equilibrium point 2 and equilibrium point 3 are complicated to be solved analytically, then they are solved numerically by Gauss Elimination Method. From each equilibrium point, it will be analyzed its stability using the eigenvalue method from the Jacobian matrix.

## 2.4. Stability analysis

From the differential equation system in equation (1), equation (2), and equation (3), they will be constructed Jacobian matrix in equation (7). Suppose:

$$f_1 = r_2 N(1 - b_2 N) - c_4 N$$

$$f_2 = r_1 T(1 - b_1 T) - \frac{\rho IT}{\alpha + T_0} - c_2 IT + c_3 N$$

$$f_3 = s + d_2 \left( \frac{\rho IT}{\alpha + T_0} \right) - c_1 IT - d_1 I$$

Then the Jacobian matrix is:

$$Jac = \begin{bmatrix} \frac{\partial f_1}{\partial N} & \frac{\partial f_1}{\partial T} & \frac{\partial f_1}{\partial I} \\ \frac{\partial f_2}{\partial N} & \frac{\partial f_2}{\partial T} & \frac{\partial f_2}{\partial I} \\ \frac{\partial f_3}{\partial N} & \frac{\partial f_3}{\partial T} & \frac{\partial f_3}{\partial I} \end{bmatrix} = \begin{bmatrix} r_2 - 2r_2 b_2 N - c_4 & 0 & 0 \\ c_3 & r_1 - 2r_1 b_1 T - \frac{\rho I}{\alpha + T_0} - c_2 I & -\frac{\rho T}{\alpha + T_0} - c_2 T \\ 0 & \frac{d_2 \rho I}{\alpha + T_0} - c_1 I & \frac{d_2 \rho T}{\alpha + T_0} - c_1 T - d_1 \end{bmatrix} \quad (7)$$

For analyzing the stability, we determine the eigenvalue from the equilibrium point on the Jacobian matrix using  $\det(\lambda I - Jac) = 0$ . The system is stable if all real of the eigenvalue are  $\lambda_1 < 0, \lambda_2 < 0, \lambda_3 < 0$  (negative) [11].

In equilibrium point 1, it is stable if  $\frac{d_1(\alpha + T_0)r_2 r_1 + c_4 \rho s + c_4 c_2 s(\alpha + T_0)}{sr_2 \rho + sr_2 c_2(\alpha + T_0) + c_4(\alpha + T_0)d_1 r_1} > 1$

In equilibrium point 2 and equilibrium point 3, substitute numerical positive equilibrium to the Jacobian matrix. The system is stable if all real of eigenvalue are  $\lambda_1 < 0, \lambda_2 < 0, \lambda_3 < 0$  (negative) [11].

### 2.5. Optimal control of tumor growth by drugs

In optimal control of tumor growth, there are drugs in the chemotherapy process as control  $u$  applied to tumor cells to reduce the number of tumor cells. However, in practice, this process also affects normal cell and immune cells so that normal cell and immune cells are also reduced. The effectiveness range  $u$  is [0,1]. Therefore the mathematical model from equation (2), (3), and (4) become the mathematical model in equation (8), (9), and (10), respectively.

$$\dot{N} = r_2 N(1 - b_2 N) - c_4 N - a_1 u N \quad (8)$$

$$\dot{T} = r_1 T(1 - b_1 T) - \frac{\rho IT}{\alpha + T_0} - c_2 IT + c_3 N - a_2 u T \quad (9)$$

$$\dot{I} = s + d_2 \left( \frac{\rho IT}{\alpha + T_0} \right) - c_1 IT - d_1 I - a_3 u I \quad (10)$$

With  $a_1, a_2, a_3$  is the rate of reducing normal cell, tumor cell, and immune cell due to drugs in chemotherapy.

For the model, the objective function which is minimized is presented in equation (11).

$$J = \int_0^T A_1 T + A_2 u^2 dt \quad (11)$$



Where weights are  $A_1 > 0, A_2 > 0$  related to the number of tumor cells and the cost of drugs in chemotherapy, respectively. From the model, the number of tumor cells and the cost of drugs in chemotherapy will be minimized. The goal is to find optimal control  $u^*$ .

### 2.6. Pontryagin's maximum principle

If  $u$  is an optimal control corresponding state system, there exist adjoint variables  $(\lambda_N \quad \lambda_T \quad \lambda_I)$  which satisfy equation (12), (13), (14), and (15) [6].

$$\dot{\lambda}_N = -\frac{\partial H}{\partial N} = -\lambda_N(r_2 - 2r_2b_2N - c_4 - a_1u) - \lambda_Tc_3 \quad (12)$$

$$\dot{\lambda}_T = -\frac{\partial H}{\partial T} = -A_1 - \lambda_T\left(r_1 - 2r_1b_1T - \frac{\rho I}{\alpha + T_0} - c_2I - a_2u\right) - \lambda_I\left(d_2\left(\frac{\rho I}{\alpha + T_0}\right) - c_1I\right) \quad (13)$$

$$\dot{\lambda}_I = -\frac{\partial H}{\partial I} = -\lambda_T\left(-\frac{\rho T}{\alpha + T_0} - c_2T\right) - \lambda_I\left(d_2\left(\frac{\rho T}{\alpha + T_0}\right) - c_1T - d_1 - a_3u\right) \quad (14)$$

$$\lambda_N(T) = 0, \lambda_T(T) = 0, \lambda_I(T) = 0 \quad (15)$$

Where the Hamiltonian is expressed in equation (16).

$$H = A_1T + A_2u^2 + (\lambda_N \quad \lambda_T \quad \lambda_I) \begin{pmatrix} r_2N(1-b_2N) - c_4N - a_1uN \\ r_1T(1-b_1T) - \frac{\rho IT}{\alpha + T_0} - c_2IT + c_3N - a_2uT \\ s + d_2\left(\frac{\rho IT}{\alpha + T_0}\right) - c_1IT + d_1I - a_3uI \end{pmatrix} \quad (16)$$

Furthermore, we can find the optimal control  $u^*$

$$\frac{\partial H}{\partial u} = 0 \quad (17)$$

$$\frac{\partial H}{\partial u} = 2A_2u + \lambda_N(-a_1N) + \lambda_T(-a_2T) + \lambda_I(-a_3I) = 0 \quad (18)$$

$$u = \min\left(1, \max\left(0, \frac{\lambda_N a_1 N + \lambda_T a_2 T + \lambda_I a_3 I}{2A_2}\right)\right) \quad (19)$$

### 2.7. Forward-backward sweep method

The forward-backward sweep method applied to optimal control of tumor growth can be designed as follows [12][9]:

Suppose state variables and adjoint variables are :

$$f_1 = rN(1-b_2N) - c_4N - a_1uN$$

$$f_2 = rT(1-b_1T) - \frac{\rho IT}{\alpha + T_0} - c_2IT + c_3N - a_2uT$$

$$f_3 = s + d_2\left(\frac{\rho IT}{\alpha + T_0}\right) - c_1IT - d_1I - a_3uI$$

$$\begin{aligned}
g_1 &= -\lambda_N(r_2 - 2r_2b_2N - c_4 - a_1u) - \lambda_Tc_3 \\
g_2 &= -A_1 - \lambda_T\left(r_1 - 2r_1b_1T - \frac{\rho I}{\alpha + T_0} - c_2I - a_2u\right) - \lambda_I\left(d_2\left(\frac{\rho I}{\alpha + T_0}\right) - c_1I\right) \\
g_3 &= -\lambda_T\left(-\frac{\rho T}{\alpha + T_0} - c_2T\right) - \lambda_I\left(d_2\left(\frac{\rho T}{\alpha + T_0}\right) - c_1T - d_1 - a_3u\right)
\end{aligned}$$

The forward-backward sweep method algorithm is as follows :

While (process has not converged yet)  $u_{old} = u$

1. Compute the solution of state variables forward with the initial condition  $N_0, T_0, I_0$  are given using Runge Kutta fourth-order.

$$\begin{aligned}
k_{1j} &= f_j(N_i, T_i, I_i, u_i), j=1,2,3 \\
k_{2j} &= f_j\left(N_i + \frac{h}{2}k_{11}, T_i + \frac{h}{2}k_{12}, I_i + \frac{h}{2}k_{13}, \frac{u_i + u_{i+1}}{2}\right), j=1,2,3 \\
k_{3j} &= f_j\left(N_i + \frac{h}{2}k_{21}, T_i + \frac{h}{2}k_{22}, I_i + \frac{h}{2}k_{23}, \frac{u_i + u_{i+1}}{2}\right), j=1,2,3 \\
k_{4j} &= f_j(N_i + hk_{31}, T_i + hk_{32}, I_i + hk_{33}, u_{i+1}), j=1,2,3 \\
N_{i+1} &= N_i + \frac{h}{6}(k_{11} + 2k_{21} + 2k_{31} + k_{41}) \\
T_{i+1} &= T_i + \frac{h}{6}(k_{12} + 2k_{22} + 2k_{32} + k_{42}) \\
I_{i+1} &= I_i + \frac{h}{6}(k_{13} + 2k_{23} + 2k_{33} + k_{43})
\end{aligned}$$

2. Compute the solution of adjoint variables backward with the final condition  $\lambda_{N(T)}, \lambda_{T(T)}, \lambda_{I(T)}$  are given using Runge Kutta fourth-order.

$$\begin{aligned}
l_{1j} &= g_j(\lambda_{N(i)}, \lambda_{T(i)}, \lambda_{I(i)}, N_i, T_i, I_i, u_i), j=1,2,3 \\
l_{2j} &= g_j\left(\lambda_{N(i)} - \frac{h}{2}l_{11}, \lambda_{T(i)} - \frac{h}{2}l_{12}, \lambda_{I(i)} - \frac{h}{2}l_{13}, \frac{N_i + N_{i-1}}{2}, \frac{T_i + T_{i-1}}{2}, \frac{I_i + I_{i-1}}{2}, \frac{u_i + u_{i-1}}{2}\right), j=1,2,3 \\
l_{3j} &= g_j\left(\lambda_{N(i)} - \frac{h}{2}l_{21}, \lambda_{T(i)} - \frac{h}{2}l_{22}, \lambda_{I(i)} - \frac{h}{2}l_{23}, \frac{N_i + N_{i-1}}{2}, \frac{T_i + T_{i-1}}{2}, \frac{I_i + I_{i-1}}{2}, \frac{u_i + u_{i-1}}{2}\right), j=1,2,3 \\
l_{4j} &= g_j(\lambda_{N(i)} - hl_{31}, \lambda_{T(i)} - hl_{32}, \lambda_{I(i)} - hl_{33}, N_{i-1}, T_{i-1}, I_{i-1}, u_{i-1}), j=1,2,3 \\
\lambda_{N(i-1)} &= \lambda_{N(i)} - \frac{h}{6}(l_{11} + 2l_{21} + 2l_{31} + l_{41}) \\
\lambda_{T(i-1)} &= \lambda_{T(i)} - \frac{h}{6}(l_{12} + 2l_{22} + 2l_{32} + l_{42}) \\
\lambda_{I(i-1)} &= \lambda_{I(i)} - \frac{h}{6}(l_{13} + 2l_{23} + 2l_{33} + l_{43})
\end{aligned}$$

3. Compute the optimal control  $u^*$  using equation (19)
4. Update optimal control

$$u \leftarrow \frac{u + u_{old}}{2} \quad (20)$$

5. Compute performance index as an objective function

$$J(u) = \sum_{k=0}^{T-1} (A_1 T(k)^2 + A_2 u(k)^2) \quad (21)$$

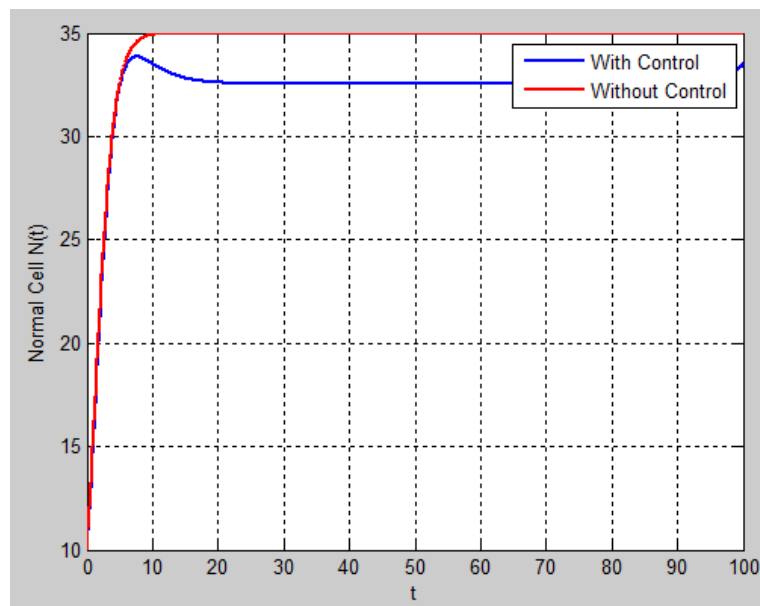
### 3. Result and discussion

It is assumed that there are between  $10^8$  and  $10^9$  cells per cubic centimeters of tissue [8]. It is important to be noted that because the amount of normal cell, tumor cell, and the immune cell is various in the type of tumor disease, then the model is only as a general model in simulation so that it is assumed dimensionless and does not require the units. Parameters used in optimal control of tumor growth simulation and initial condition can be seen in table 2.

**Table 2.** Simulation parameters.

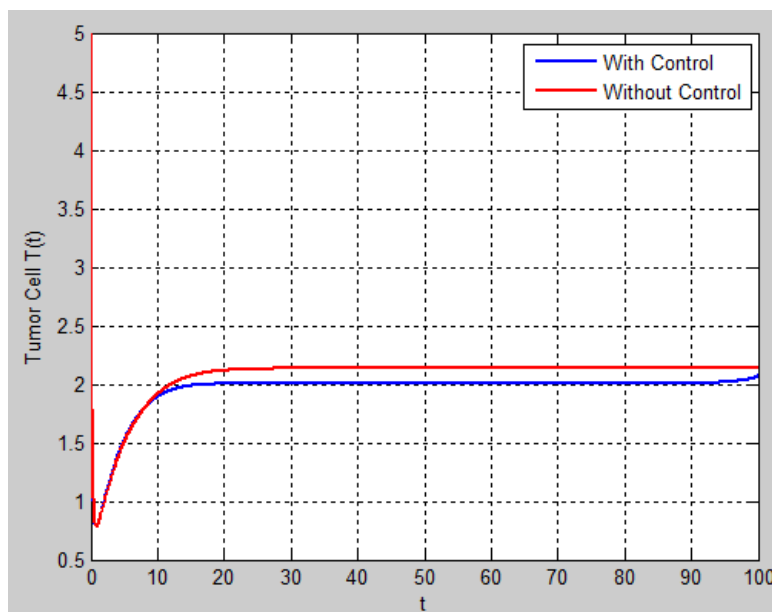
Parameters	Value
The population of normal cell $N(0)$	10
The population of tumor cell $T(0)$	5
The population of immune cell $I(0)$	10
Intrinsic rate (per capita growth rate) of tumor cell growth $r_1$	1
Intrinsic rate (per capita growth rate) of normal cell growth $r_2$	1
Carrying capacity of the tumor cell population $b_1$	1/2
Carrying capacity of the normal cell population $b_2$	1/50
Search rate of tumor cell by the immune cell $\rho$	0.01
Conversion factors $d_2$	0.5
The natural death rate of immune cell $d_1$	0.2
Immune threshold rate $\alpha$	0.3
The growth rate of immune cell (constant) $s$	5
Coefficient of an inactive immune cell due to interaction with tumor cell $c_1$	0.2
Coefficient of dead tumor cell due to interaction with immune cell $c_2$	0.6
The rate of increasing tumor cell due to normal cell mutation to tumor cell $c_3$	0.3
The rate of decreasing normal cell due to normal cell mutation to tumor cell $c_4$	0.3
Weight-related to the amount of tumor cell $A_1$	1
Weight-related to the cost of drugs in chemotherapy $A_2$	5

Figures 1, 2, and 3 show the numerical solution of a normal cell, tumor cell, and immune cell with and without drugs in chemotherapy as control, respectively. From the graph, the effect of drugs in chemotherapy can reduce the number of normal cells in figure 1, the number of tumor cells in figure 2, and the number of immune cells in figure 3. The value of the rate of reducing normal cells due to drugs in chemotherapy  $a_1$  is 0.2, the rate of reducing tumor cell due to drugs in chemotherapy  $a_2$  is 0.8, and the rate of reducing immune cell due to drugs in chemotherapy  $a_3$  is 0.2.

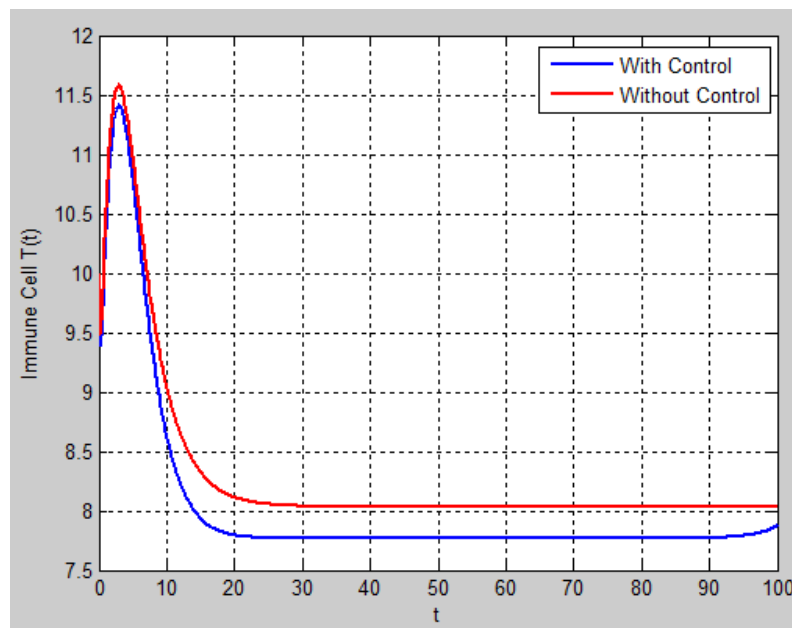


**Figure 1.** Numerical solutions of tumor growth of a normal cell.

In normal cells without control, the simulation is stable. Its growth follows the logistic function for preventing the blow-up population. When drugs in chemotherapy are applied, they affect normal cell so that normal cell is reduced. In tumor cells without control, the simulation is stable. Its growth decreases because tumor cells are eaten by immune cells, then it increases because of mutation from a normal cell to a tumor cell. When drugs in chemotherapy are applied, they devastate part of the tumor cell population so that the tumor cell is reduced. In immune cells without control, the simulation is stable. The growth increases because immune cells eat tumor cells so that they duplicate their selves for attacking tumor cells. Then immune cell decreases because there are dead immune cells. When drugs in chemotherapy are applied, they affect immune cell so that immune cell is reduced.

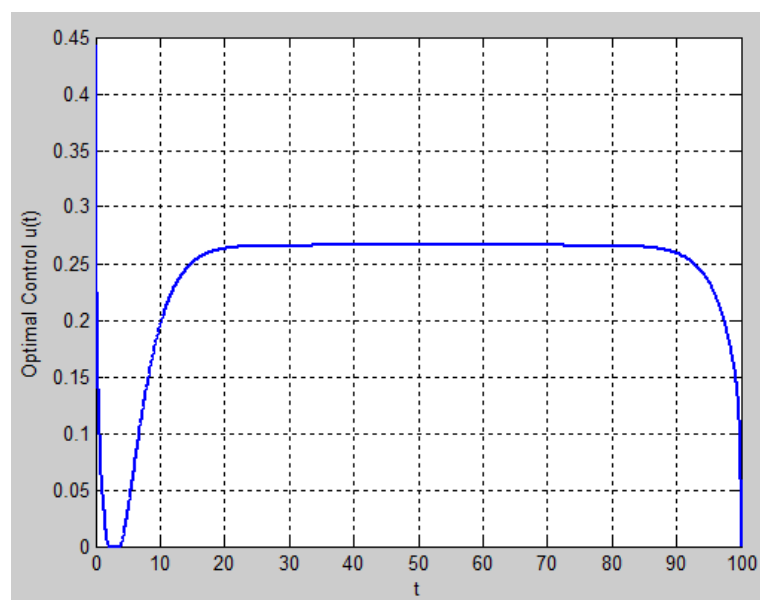


**Figure 2.** Numerical solutions of tumor growth of tumor cell.



**Figure 3.** Numerical solutions of tumor growth of the immune cell.

Figure 4 shows the control function of drugs in chemotherapy. The control function has the interval of effectiveness between 0 to 1 with 0 represents control functions fail (not effective in the whole population), and 1 represents control functions are a success (effective in the whole population).



**Figure 4.** Optimal control of drugs in chemotherapy.

#### 4. Conclusion

In the tumor growth dynamical model, there are normal cells, tumor cells, and immune cells. From the mathematical model of tumor growth, there are some equilibrium points that will be analyzed their stability using eigenvalue. In this research, from the mathematical model of tumor growth, it will be added control, i.e., drugs in chemotherapy. The method used for solving optimal control problems and resulting numerical solutions is Forward Backward Sweep Method. Based on simulation results, drugs

in chemotherapy give effects in a normal cell, tumor cell, and immune cell. The development of this research is optimizing the weights of the performance index so that the performance index is more optimal.

## References

- [1] Rahmalia D and Herlambang T 2017 *Proc. The 7<sup>th</sup> Annual Basic Sci. Int.l Conf.* (Malang: BaSIC)
- [2] Rahmalia D and Herlambang T 2018 *Int. J. Comput. Sci. Appl. Math.* **4** 27-31
- [3] Rahmalia D 2015 *Unisda J. Math. Comput. Sci.(UJMC)* **1** 11-9
- [4] Rahmalia D and Rohmatullah A 2019 *J. Ilm. Soulmath: J. Edukasi Pendidik. Mat.* **7** 159-73
- [5] Murray J D 2002 *Mathematical Biology an Introduction* (New York: Springer)
- [6] El Hia M, Balatif O, Rachik M and Bouyaghroum J 2013 *Int. J. Comput. Sci. Issues* **10** 230-6
- [7] Mu'tamar K, Rahmalia D and Sutimin S 2019 *Eksakta: J. Ilmu-ilmu MIPA* **19** 128-42
- [8] De Pillis L G and Radunskaya A 2003 *Math. Comput. Model.* **37** 1221-44
- [9] Rahmalia D, Herlambang T, Rohmah A M and Muhith A 2020 *J. Phys.: Conf. Ser.* **1594** 012040
- [10] Burl J B 1999 *Linear Optimal Control* (Menlo Park, CA: Addison-Wesley)
- [11] Brauer F and Castillo-Chavez C 2012 *Mathematical Models in Population Biology and Epidemiology* (New York: Springer)
- [12] Lenhart S and Workman J T 2007 *Optimal Control Applied to Biological Models* (London: CRC Press).